TABLE II
REFLECTION AND TRANSMISSION COEFFICIENTS AND ABSORPTION FOR $\varepsilon_r = 30$, $\tan\delta = 0.6$, $D = 5.0$ mm, $l = 15.0$ mm AS A FUNCTION OF FREQUENCY

Frequency (GHz)	Reflection Coefficient	Transmission Coefficient	Fraction Absorbed
8	$0.844 \angle -172.6^\circ$	$0.0058 \angle 1.08^\circ$	0.288
9	$0.778 \angle -173.4^\circ$	$0.0044 \angle -153.4^\circ$	0.395
10	$0.727 \angle -168.3^\circ$	$0.0079 \angle -27.9^\circ$	0.471
11	$0.617 \angle -176.1^\circ$	$0.0096 \angle 157.42^\circ$	0.619
12	$0.544 \angle -175.9^\circ$	$0.0118 \angle -169.3^\circ$	0.704

TABLE III
REFLECTION AND TRANSMISSION COEFFICIENTS AND ABSORPTION FOR THE SLAB OF TABLE II AT 9.0 GHz AND A VARIETY OF DIELECTRIC CONSTANTS

ε_r	Reflection Coefficient	Transmission Coefficient	Fraction Absorbed
21	$0.794 \angle -168.4^\circ$	$0.0126 \angle -82.7^\circ$	0.369
30	$0.778 \angle -173.4^\circ$	$0.0045 \angle -153.4^\circ$	0.395
51	$0.801 \angle 179.8^\circ$	$0.0026 \angle 130.36^\circ$	0.358

power absorbed for this lossy slab are given in Table II for several frequencies and in Table III for several different dielectric constants at 9 GHz. The fraction of incident power absorbed is more sensitive to frequency than to relative dielectric constant. The absorption at 9.0 GHz and $\varepsilon_r = 30$ (39.4 percent) compares favorably with that observed in real pupae (32 percent) [10].

V. DISCUSSION

The agreement between theory and experiment which is displayed in Figs. 2 and 3 and in Table I leads us to believe that the five-term field approximation technique will be adequate for a variety of applications.

For the cases examined here, maximum heating always occurred at the front surface of the slab and no significant secondary maxima were observed. We believe, however, that other combinations of slab parameters could well result in significant standing waves along the slab and a resultant inhomogeneity of heat input.

The effect of partial filling in the height of the guide was not examined due to its complexity and due to the agreement between the power absorption calculated for a full-height slab and that observed for actual pupae. We note, however, that the calculated and observed field distributions for this configuration do differ significantly and caution therefore against the uncritical application of these techniques to partial-height obstacles.

APPENDIX

The normalization factor W_m for function $g_m(x)$ in (5) is given by

$$W_m = \left[\frac{1}{|k_{1m}|^2} \left\{ \frac{\sin (k_{1m} - k_{1m}^*)d}{k_{1m} - k_{1m}^*} \quad \frac{\sin (k_{1m} + k_{1m}^*)d}{k_{1m} + k_{1m}^*} \right\} + \frac{|\cos k_{1m} d|^2}{\left| k_{2m} \sin \frac{k_{2m} D}{2} \right|^2} \cdot \left\{ \frac{\sin (k_{2m} - k_{2m}^*) \frac{D}{2}}{k_{1m} - k_{1m}^*} + \frac{\sin (k_{2m} + k_{2m}^*) \frac{D}{2}}{k_{2m} + k_{2m}^*} \right\} \right]^{-1/2}. \quad \text{(A1)}$$

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Scattering of Microwaves by Dielectric Materials Used in Laboratory Animal Restrainers

JAMES C. LIN AND CHUAN-LIN WU, STUDENT MEMBER, IEEE

***Abstract*—In most experimental investigations of the biological effects of microwave radiation, it is necessary to use low-loss dielectric materials for restraining animals under irradiation. Because of the complexity of the analysis of the animal–restrainer combination, an analysis is made of the scattering of microwave fields by a simplified model of the restrainer with no animal present. The model chosen is that of a plane wave incident at an arbitrary angle upon a rectangular slab of finite width and thickness. Numerical results indicate that the scattered fields within a square region of one wavelength in distance from the slab surfaces are greatly enhanced and highly nonuniform. In particular, the maxima for parallel incidence exceed those for normal incidence by almost a factor of 2.**

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The authors are with the Department of Electrical Engineering, Wayne State University, Detroit, MI 48202.

Introduction

Recent emphasis on the biological effects and health hazards of nonionizing electromagnetic radiation has been on the quantitative relationship between observed biological effects and the physical variables of electromagnetic radiation [1]–[3]. The observed biological effects are necessarily related to the incident power density and the field generated inside the biological system. Therefore a carefully executed microwave biological-effect experiment must take into consideration the constituency, geometry and size of the subject, and the microwave source configuration, frequency and location.

Since most experiments involve the use of animals and tissue samples of various species and types, it is necessary to incorporate into these experimental protocols low-loss dielectric materials for purposes of restraining animals and holding samples. The incident microwave field is complicated by such things as scattering and refraction. Consequently, the total exposure experienced by the restrained laboratory subject may be considerably different from that experienced in the absence of such foreign materials.

However, the analysis of the exposure actually experienced by an animal in a restrainer is obviously very difficult. Accordingly, as a first indication of the effects of the restrainer on the power absorbed by a restrained animal, an analysis of the fields scattered by a simplified model of the restrainer alone is presented. Although one must be very careful in drawing conclusions about the effect of the restrainer on the power absorbed by an animal from the calculations of the fields scattered by the restrainer in the absence of the animal, the nature of these scattered fields can give some indication of the effect of the restrainer. For example, if the animal were to be located in a region where the scattered fields of the isolated restrainer are very weak, one would expect that the restrainer would probably have less effect on the power absorbed by the animal than if the animal were to be located in a region where the scattered fields of the isolated restrainer are very strong. The interaction between the animal and the restrainer would have to be taken into account, of course, to find the actual effect, but a knowledge of the scattered fields should still be helpful as a first indication of how to design the restrainer, even though the only sure way to minimize the effects of the restrainer is to construct one and measure the effects.

To provide this information about the scattered fields produced by the isolated restrainer, an analysis is presented in this short paper using simplified models of the restrainer, rectangular Plexiglas, and soda-borosilicate glass slabs of thickness and width commonly found in animal restrainers used in microwave biological-effects research. The incident microwave radiation is assumed to be a plane wave, and the slabs are taken to be infinitely long. The method of solution employed closely follows one previously developed [4], [5]. Although there is no restriction on the direction of the incident plane wave, the scattered fields are evaluated for 90 and 180° angles of incidence.

Method of Solution

Consider the rectangular dielectric slab shown in Fig. 1; a plane wave with exp $(j\omega t)$ is incident on it. The electric field is polarized along the axis of the slab. A general formulation has been developed [4], [5] which gives the induced field inside a dielectric cylinder of arbitrary cross section and the associated scattering patterns. The technique is based on an integral equation approach. The dielectric cylinder is divided into square cells which are small enough such that the electric field in each cell is almost uniform. A system of linear equations is obtained by enforcing at the center of each cell the condition that the total field must be equal to the sum of the incident and scattered fields. This system of equations is then inverted by known matrix techniques to give the electric field in each cell. The scattered field at any other point in space can be calculated by integrating the induced fields over the cross section of the cylinder.

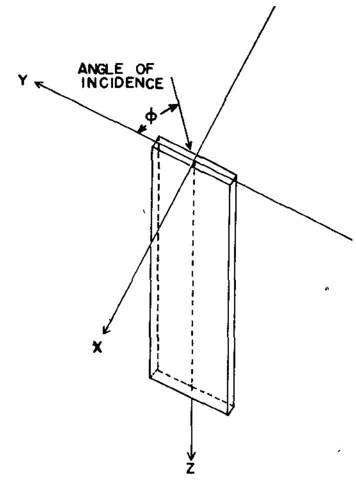

Fig. 1. Section of infinitely long rectangular dielectric slab of finite width and thickness.

This technique has a number of advantages. The results approach the exact solution if a sufficiently large number of cells are employed. Solutions for arbitrary shape and composition can be obtained systematically and efficiently by using a large modern digital computer. In this short paper we are interested in the numerical results obtained by applying the aforementioned technique and the implications these results have on microwave biological-effects research. Therefore, although we will not detail the theory given explicitly in [4] and [5], we will present a brief summary.

The total field E in the presence of the dielectric slab is given by the sum of the incident and scattered fields, i.e.,

$$E = E^i + E^s. \qquad (1)$$

The scattered field E^s is generated by an induced current J inside the slab, given by

$$J = j\omega\varepsilon_0(\varepsilon - 1)E \qquad (2)$$

where ε_0 is the free-space permittivity and ε is the complex relative dielectric constant of the slab.

If we divide the cross section of the dielectric slab into cells which are sufficiently small so that the electric field strength is essentially uniform over the cell, and moreover, if we replace the rectangular cells with equivalent circular cells of the same cross-sectional area, comparatively simple expressions can be derived for the scattered field. Since, for μ_0 equal to the free-space permeability, the scattered field due to a current element $dI = Jds$ parallel to the z axis is

$$dE^s = -(\omega\mu_0/4)H_0^{(2)}(k\rho)\, Jds \qquad (3)$$

thus

$$E^s(xy) = (-jk^2/4) \iint (\varepsilon - 1)E(x',y')H_0^{(2)}(k\rho)\, dx'\, dy' \qquad (4)$$

where (x,y) and (x',y') are the coordinates of the observation point and the source point, respectively, $k = 2\pi/\lambda$, λ is the free-space wavelength, $H_0^{(2)}(k\rho)$ is the Hankel function of the second kind and zeroth order, and ρ is the distance from the current element to the observation point. Equation (4) is valid for the scattered field at any point inside or outside the dielectric region. If we now enforce the condition set forth by (1) at the center of cell m, a set of equations is obtained such that

$$E_m + (jk^2/4) \sum_{n=1}^{N} (\varepsilon_n - 1)E_n \iint_{\text{cell } n} H_0^{(2)}(k\rho)\, dx'\, dy' = E_m^i, \qquad m = 1,2,\cdots,N \quad (5)$$

where ε_n and E_n are the complex relative dielectric constant and the electric field strength at the center of cell n, respectively, N represents the total number of cells, and

$$\rho = [(x' - x_m)^2 + (y' - y_m)^2]^{1/2}. \quad (6)$$

As indicated previously, if a circular region is taken for the cell, the integral in (5) involving the zeroth-order Hankel function reduces to a simple expression and the set of linear equations in (5) can be represented by

$$\sum_{n=1}^{N} C_{mn}E_n = E_m^i, \qquad m = 1,2,\cdots,N. \quad (7)$$

If a_n stands for the radius of the circular cell, the elements of the C_{mn} matrix become

$$C_{mn} = 1 + (\varepsilon_n - 1)(j/2)[\pi k a_n H_1^{(2)}(ka_n) - 2j], \qquad n = m \quad (8)$$

$$C_{mn} = (j\pi k a_n/2)(\varepsilon_n - 1)J_1(ka_n)H_0^{(2)}(k\rho_{mn}), \qquad n \neq m \quad (9)$$

with

$$\rho_{mn} = [(x_m - x_n)^2 + (y_m - y_n)^2]^{1/2}. \quad (10)$$

Since (4) is valid for the scattered field at any point, the scattered field outside the slab dielectric under the same conditions stated previously is given by

$$E^s(x,y) = -j(\pi k/2) \sum_{n=1}^{N} (\varepsilon_n - 1)E_n a_n J_1(ka_n)H_0^{(2)}(k\rho_n) \quad (11)$$

where ρ_n is the distance from the center of cell n to the observation point (x,y).

The scattered field at large distances from the scattering object can be obtained by using the asymptotic form for the Hankel function of large argument and taking

$$\rho_n = \rho_0 - x_n \cos\phi - y_n \sin\phi \quad (12)$$

where ρ_0 and ϕ are the polar coordinates of the distant observation point.

The scattered far field is then given by

$$E^s(\rho_0,\phi) = -j(\pi k/2)(2j/\pi k\rho_0)^{1/2} \sum_{n=1}^{N} (\varepsilon_n - 1)E_n a_n J_1(ka_n) \cdot \exp[-jk\rho_n]. \quad (13)$$

Numerical Results and Discussion

The scattered fields were calculated for Plexiglas and soda-borosilicate glass slabs with the aid of an IBM 360/50 computer. The computer program was checked against known results in the literature [4]; in all cases there was complete agreement. The complex relative dielectric constants are given in Table I. The thickness and width of the slabs were 0.05λ and 1.05λ, respectively. At 2450 MHz, a frequency often used for biological-effects research, these values correspond to 0.006 and 0.13 m or 0.25 and 5 in. The slab was divided into 21 subsections along the width of the slab for the internal fields. A 1-V/m incident field is assumed.

TABLE I
Complex Relative Dielectric Constants

Material	Complex Relative Dielectric Constant
Plexiglas	2.60 - j 0.0016
Soda-B Glass	4.82 - j 0.0300

TABLE II
Scattered Fields at a Distance of One Wavelength in Both x and y Directions from the Center of the Slabs

Dielectric Material	incidence angle	Near Field Formulation		Far Field Formulation	
		Magnitude	Phase	Magnitude	Phase
Plexiglas	90°	0.068986	158.07	0.064986	145.61
Plexiglas	180°	0.205675	139.82	0.209235	146.40
Soda-B Glass	90°	0.103041	153.92	0.086158	131.00
Soda-B Glass	180°	0.517295	106.08	0.530773	113.21

The field scattered by the dielectric slabs can be calculated at any point outside by means of (11) and (13). For distances up to 1.5λ, (11) was used and the distant field distribution may be evaluated using (13). Table II indicates that at a distance of one wavelength in both x and y directions the scattered fields given by (11) and (13) are within 6 percent of each other except in the normal incidence case for glass. Therefore for distances greater than one wavelength, which is equivalent to 0.123 m at 2450 MHz, it is sufficient to evaluate the scattered field strength using the far-field formulation of (13).

The scattered fields in the vicinity of both Plexiglas and glass were evaluated using (11) for a wide range of incident angles. Some typical results are shown in Figs. 2–5 for 90 and 180° angles of incidence. These results are for distances of 0.026λ (0.003 m at 2450 MHz) to 1.5λ (0.18 m) from the surface of the dielectric materials. The electric field is polarized in the positive z direction and the cross-sectional area of the slabs is in the xy plane (Fig. 1). As expected, the scattered field distributions are symmetric for normal incidence (90°) between the front and back of the dielectric slab and about the center of the slab on both sides of the x axis (Figs. 2 and 4). Although the scattered field is highly nonuniform in the immediate vicinity of the slab and approaches a cosine distribution, it decreases rapidly and becomes nearly uniform approximately one wavelength from the slab. The scattered field distribution for a parallel incident wave (180°) is symmetric about the broad face of the slab and asymmetric about the x axis. The maxima of the scattered field in this case occur near the center of the slab but away from the incident plane wave. Moreover, the maxima for parallel incidence are greater by a factor of 2 than those for normal incidence. The scattered field in the shadow region decreases much more slowly than in other directions: it reduces to 1 percent of the applied radiation about five wavelengths away. It should be mentioned that for incident angles greater than 90° but smaller than 180° the induced and scattered fields for both Plexiglas and soda-borosilicate glass are vector sums of the corresponding normal and parallel incidence cases and, in general, smaller than those for parallel incidence.

It should also be mentioned that related studies have shown that the scattered field for the parallel incidence case is nearly proportional to the width of the slab, while the scattered field for normal incidence remains practically unchanged for a given dielectric slab. The scattered field is also a function of the thick-

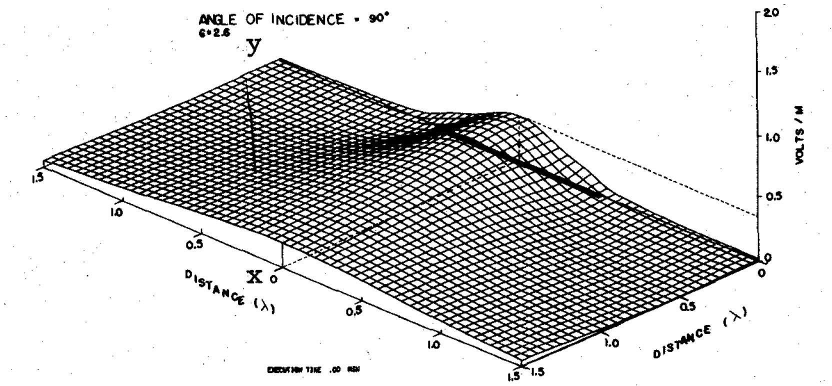

Fig. 2. Scattered field in the vicinity of a Plexiglas slab with a normally ($\phi = 90°$) incident plane wave. The orientation is the same as shown in Fig. 1 and the slab cross section is indicated by the dark heavy line.

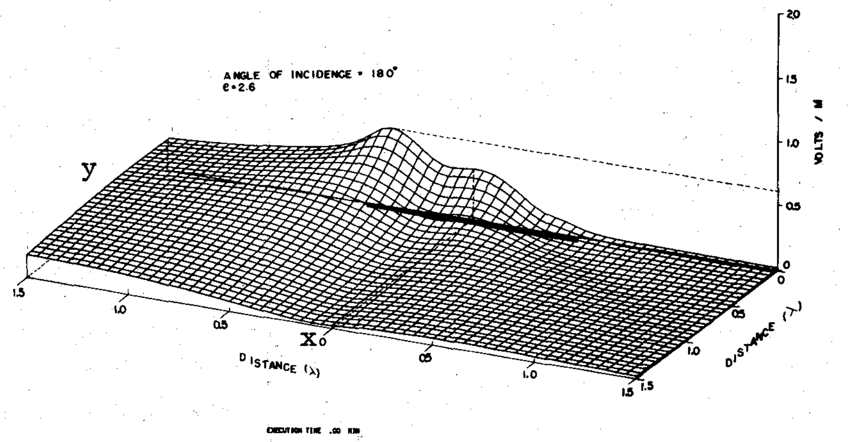

Fig. 3. Scattered field in the vicinity of a Plexiglas slab by a parallel ($\phi = 180°$) incident plane wave. The orientation is the same as shown in Fig. 1 and the slab cross section is indicated by the dark heavy line.

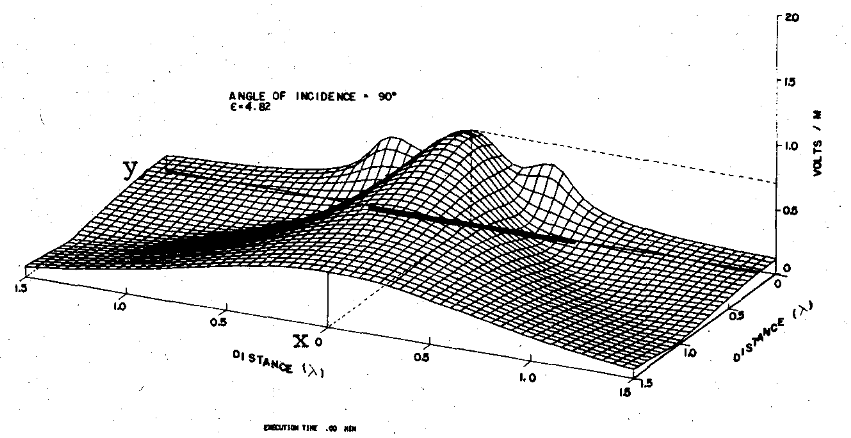

Fig. 4. Scattered field in the vicinity of a soda-borosilicate glass slab with a normally ($\phi = 90°$) incident plane wave. The orientation is the same as shown in Fig. 1 and the slab cross section is indicated by the dark heavy line.

ness and composition of the dielectric slab. In general, the thinner the slab and the closer the dielectric constant to that of free space, the smaller the perturbation due to scattering contributions as would be expected.

Conclusions

The scattered fields outside rectangular low-loss dielectric slabs of finite width and thickness have been studied for both normal and parallel incidences. These numerical results are of interest in investigations of biological effects of microwave radiation. In particular, they serve to indicate the perturbation produced by low-loss dielectric materials commonly used for animal holding and restraining purposes.

The results for scattered field in the vicinity of the dielectric slabs indicate nonuniform and enhanced field strength for distances less than one wavelength away from the surface of the dielectric material. The maxima of the scattered field of a plane wave incident at 180° occur, for both Plexiglas and glass, near

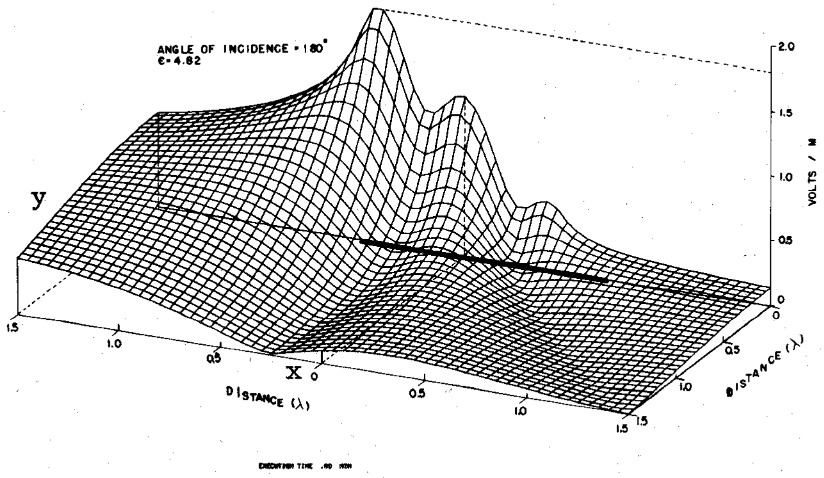

Fig. 5. Scattered field in the vicinity of a soda-borosilicate glass slab by a parallel ($\phi = 180°$) plane wave. The orientation is the same as shown in Fig. 1 and the slab cross section is indicated by the dark heavy line.

the center but shifted toward the back of the dielectric slabs in contrast to the normal incidence case where maxima occur at the point closest to the center of the slabs. The maxima of the scattered fields for parallel incidence are generally greater than those of normal incidence. This is in variance to some conjectures that when a Plexiglas wall lies parallel to the direction of propagation of a plane wave it creates only a slight field perturbation relative to that created by other wall orientations.

Keeping in mind the caution necessary in interpretation of the results that were mentioned in the Introduction, the results of this short paper suggest that for minimal perturbation the broad face of an animal holder should be oriented toward the direction of propagation. Moreover, the individual members of the restraining device should be as thin as practicable and the dielectric materials should be chosen to correspond as much as possible to the characteristics of free space. It is also clear that for the least amount of perturbation on both the incident and induced fields, it is necessary to place the animal in a region where the scattered fields due to restrainer alone are insignificant. This region is generally located in the forward portion of a restraining device. For an animal placed in the aft portion of the restraining device, the power absorbed could be altered significantly for parallel incidence because the field scattered by the isolated restrainer is strong in that region. However, the precise amount of perturbation will depend on the material used and the shape and size of the restraining device, as well as the animal involved, and is beyond the scope of this short paper. It should be emphasized that at the moment it is impossible to expose animals without some degree of restraining.

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Bidirectionality in Gyrotropic Waveguides

PAUL R. McISAAC, MEMBER, IEEE

***Abstract*—A waveguide containing gyrotropic media is shown to be bidirectional if it possesses at least one of the following symmetry operations: reflection in a plane perpendicular to the waveguide axis; 180° rotation about an axis perpendicular to the waveguide axis; or inversion in a point on the waveguide axis. The relation between the modal electromagnetic field components for any complementary mode pair is given for each symmetry case.**

I. Introduction

The modes of a uniform waveguide have electromagnetic fields which vary longitudinally as exp $(-\gamma_n z)$, where γ_n is the propagation constant for the nth mode and z is the waveguide axis. For a fixed frequency the mode spectrum of a uniform waveguide is infinite. If the waveguide's transverse boundaries are closed (for example, perfect conductors), there is an infinite discrete spectrum. For all uniform waveguides of current technical interest whose transverse boundaries are open (the fields extend to infinity), there are a finite number of discrete modes plus a continuous spectrum. A uniform waveguide is termed bidirectional if, at any frequency, for each mode with propagation constant γ_n there exists a complementary mode with propagation constant $-\gamma_n$.

It is well known that if the media filling a uniform waveguide have permeability and permittivity dyadics which are symmetric ($\tilde{\mu} = \mu$, $\tilde{\varepsilon} = \varepsilon$), the waveguide will be both bidirectional and reciprocal. However, the absence of media with nonsymmetric permeability and permittivity dyadics is only a sufficient and not a necessary condition for bidirectionality. Under some conditions waveguides containing media with nonsymmetric permeability and/or permittivity dyadics, such as gyrotropic media, may be bidirectional. This short paper establishes sufficient conditions for bidirectionality in uniform waveguides containing gyrotropic media. The discussion applies to inhomogeneous, lossy, uniform waveguides with open or closed boundaries.

Let S be a generalized medium susceptibility which relates a

Manuscript received February 6, 1975; revised October 6, 1975.
The author is with the School of Electrical Engineering, Cornell University, Ithaca, NY 14853.